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On the cumulative human population of the earth

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ON THE CUMULATIVE HUMAN POPULATION OF THE EARTH

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-oOo-

ABSTRACT

The study of human population growth has been a field of active research for many years. While a great amount of this research has been to project future population patterns, this study is concerned rather with describing and quantifying populations which have or do exist in order to answer the much asked question - "how many people have ever lived?".

Firstly, the relevant history of man from when he first walked the earth until the present is studied in an attempt to estimate, as reliably as possible, the population size at different points in man's development. These estimates closely resemble those of Carr-Saunders, Willcox and the United Nations from 1650 to the present, but prior to this are different from previous studies.

Models have been constructed to simulate the trends discovered in the above estimates and are discussed in the categories of: pre-agricultural, agricultural and industrial societies. From these models, the cumulative population of each period has been calculated.

Finally, the results of this study are analysed with a view to determining the feasibility of long run population growth projections.

INTRODUCTION

CENSUS OR SAMPLE?

This basic question is one which faces the statistician when designing any experiment involving the collection of data. Normally this problem is under the control of the demographer. However, in the measurement of human population even the most modern methods can only supply a sample, admittedly very large, with which the behaviour of the total population can be approximated. Modern "censuses" of population began in Canada in 1665, and Iceland in 1703, and by 1801 had occurred in Sweden, Denmark, United States and Great Britain (Woytinsky p. 32), but further progress was slow, and even today no "census" figures are available for China. This situation appears even more unusual when evidence exists that population "censuses" were being taken in China and Babylon as far back in time as 3000 B.C. (Ibid. p. 32). The explanation for this reluctance is, in part at least, the unmistakeable fact that historically the taking of a "census" has invariably preceded higher taxes, conscription, confiscation of property or some other event equally unpleasant for the populace as a whole. As a direct consequence of this, records were destroyed whenever the chance arose and today, except in very few cases, little data concerning these ancient "censuses" survives.

These problems notwithstanding, it is clear that from the beginning of human history man has attempted to gain some idea of his numbers and following the dismal predictions of Thomas Malthus some projection of the future population size. In the last 100 years a great amount of research has occurred regarding the population size in previous times and this research can best be reviewed in three sections: namely hunting and gathering societies, agricultural based societies and agricultural/industrial based societies. These reviews form the bases of Chapters II, III and IV.

The population analysis of Thomas Malthus in 1798 could be described as the first mathematical model formulated. However, it is generally agreed that the models of A. J. Lotka (1907) and Sharpe and Lotka in 1911 represent the first real mathematical models in this field.

Since that time many models, both deterministic and statistical, have been postulated to analyse a myriad of questions concerning the behaviour of human population, but to date the accurate prediction of the world's future population continues to elude even the most sophisticated models.

Whatever the problem, the study of human population has an undeniable fascination, and it is hoped that the reader will find this thesis no exception.

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THE EMERGENCE OF MAN

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THE EMERGENCE OF MAN

"Man is 3.75 million years old." This headline appeared above an article in the Australian newspaper which reported the discovery of fossil bones by Dr. M. Leakey in Tanzania and Dr. D. C. Johanson in Ethiopia. But does the discovery of a hand or skull resembling that of modern man prove his existence for such a long period? As Clark (p.17) states "the physical similarities between men and anthropoid apes are so close as to leave no reasonable doubt of their near zoological affinities" and this suggests that if the above question is to be answered, the differences rather than the similarities between ape and man need to be considered. The differences can be categorised as follows; ability to walk in an erect position, brain size, dentition, ability to communicate by means of speech and reproductive capabilities of closely related species.

There seems little doubt that the first of the characteristics to emerge was the adoption of an erect posture, and although it is difficult to decide why these creatures left the trees and began travelling on the ground it is known that this transition occurred during the Miocene geological period which was characterised by extensive changes of climate in many parts of the world and as Linton (p. 6) suggests it may well have been the trees leaving our ancient ancestors rather than our ancestors leaving the trees. Whatever the reason, however, the fossils discovered by Dart in Bechuanaland in 1924 and named Australopithecus Africanus and those discovered by Broom in 1936 at Sterkfontein and named Australopithecus transvaalensis prove that this transition did occur and although some disagreement still exists as to the age of these fossils, it appears that they are from the lower or middle Pleistocene period. (Kroeber p.93). These creatures walked erect and, although as Kroeber(p.92) points out their skull capacity of a maximum of around 650 c.c. limits their brain size to that of an adult ape, the formation of their teeth was closer to human than simian. Also the contour of the forehead and upper facial area as well as the mastoid process clearly illustrate a tendency toward a developing hominid creature.

Kroeber (p.93) suggests that these creatures appeared too late to be an ancestor of modern man and represent a third evolutionary line parallel with men and gorilla-chimpanzee. Clark (p. 19) however, suggests that they were just the antecedents that might have been expected for the Pithecanthropians of the succeeding Middle Pleistocene era. Both of these points of view may well be true, as it seems unlikely that these groups appeared and then disappeared before the emergence of the next link in the evolutionary chain. As Brierley (p.27) points out, man at this stage of his development was the amateur of the animal kingdom and consequently interbreeding of the many closely related species of developing man was possible without the fertility of the offspring being affected. When this is coupled with the ruthless automatic pruning process(Brierley p.35) of natural selection and the everchanging climate forcing migration to, and conquest of, new territories the parallel development of the Pithecanthropians and their final superiority over the competing groups appears feasible.

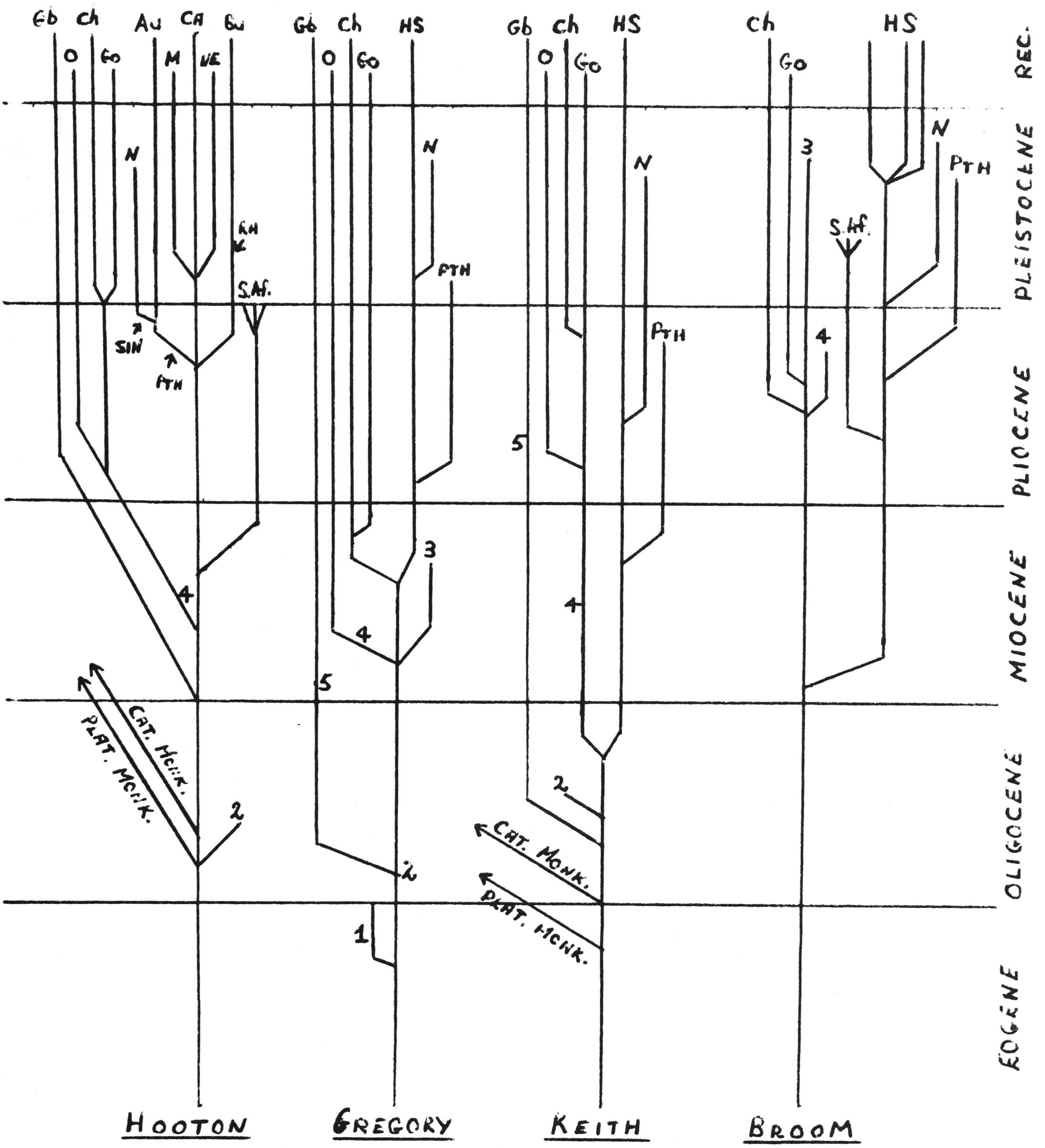
The Pithecanthropus erectus was first discovered in the Trinil beds of Java and although considered definitely hominid (Kroeber p. 83) represents the least developed known example of a true ancestor of modern man. From fossils of this type discovered near Peking it has been found that not only did these creatures walk erect, but also considerable changes had occurred in brain size. In this regard their mean skull size of 1,075c.c. places them between the Australopithecus (400 - 700 c.c.) and homo sapiens (c. 1,350 c.c.). The teeth, however, whilst being definitely hominid in character, still showed simian characteristics as did the shape of the forehead and upper facial area. As Clark (p.21) and Kroeber (p. 87) point out, however, Pithecanthropic man was certainly manufacturing crude tools and capable of using fire. Kroeber (p. 225) believes that it is at this time that language began to develop and simple speech occurred. Anthropologists agree that language is a necessity to developing culture if not even a pre-requisite, and the discovery of the fossils of some forty Pithecanthropic men near Peking suggests that they had developed some type of primitive culture at this time. This discovery would tend to support Kroeber's proposition. Since the first discovery of Pithecanthropic man in Java, fossilised remains of this species have been found which suggest the species were in Africa and Europe some 500,000 - 1,000,000 years ago. (Clark p. 20)

At the present time Pithecanthropic man is considered the last of the Protoanthropic forms, although remains of Solo man discovered near Trinil in Java are widely considered as being an example of an evolved Pithecanthropic man.

With the coming of Palecanthropic forms the fossils show the existence of a hominid creature very similar to modern man. Neandertal man was first discovered in Western Europe and the fossilised remains dated at the time of the Würm Glaciation period. However, later finds in central Europe have shown the existence of Neandertal man in earlier Riss-Würm Interglacial period. Of some interest is the tendency for the older Neandertal fossils to be closer to modern man than those found in Western Europe. One possibility, as Kroeber (p. 97) points out, is that a sub-race may have developed in Western Europe in order to survive the oncoming ice age. The discovery of what has been called Palestinian man would support this view, as Palestinian man was apparently being modified at this time in the opposite direction - i.e. toward Neanthropic man. It is certain that Neandertal man had a developed culture and apart from tool making and the use of fire had developed to the stage where he buried his dead.Skull capacity was greater in fact than that of modern man, although the forehead was still back sloping. Another variant of Neandertal man to appear at this time was Rhodesian man, who although more advanced in some areas than the Neandertal man, was considerably behind in other areas. It seems clear at this point that the ancestors of man were still evolving and vanishing according to the process of natural selection. With the coming of the Neanthropic races, however, this was to change and man, "amateur" of the animal world, was to become a specialist. Of the Neanthropic races, Cro-Magnon is perhaps the best known, but others like Afalou, Boskop, Elmenteita, Upper Choukoutieu, to name a few can be shown as ancestors of various living races today.(Kroeber p. 110) From this time, evolutionary changes in man began to slow down, and with the emergence of Homo sapiens man began to modify, rather than be modified by, his environment.

One fact appears certain from these discoveries, and that is that man as we know him today did not appear on the earth overnight, but came as a result of interbreeding of species, many of which, if not all, are extinct today. Four such possible paths of evolution are shown in Table 1.1 (Kroeber p. 55).

TABLE 1.1



HS Homo sapiens, living man: Au, Ng, Ca, Mo, Bu Australian, Negro, Caucasian, Mongoloid, Bushman races. Living Apes: Gb, O, CH, GO: Gippon, orang, chimpanzee, gorilla. Extinct man: N Neandertal, PTH Pithecanthropus, Rh Rhodesian. Extinct Apes: 1. Parapithecus, 2. Propliopithecus, 3. Swapithecus, 4. Dryopithecus, 5. Rhopithecus, - S. Af. South African forms Australopithecus, Paranthropus, Plesianthropus.

Table 1.2 summarises the classification of Hominidae (Clark p.21)

Table 1.2

GEOLOGICAL PERIOD	GENUS	SUB-GENUS	SPECIES
Late Pleistocene	Homo		H. Sapiens
			H. Neanderthalensis
			H. Rhondensiensis
Middle Pleistocene	Pithecanthropus		P. Heidelbergensis
			P. Africanus
			P. Pekinensis
			P. Javanensis
		(Telanthropus	
Early Pleistocene	Australo-pithecinae	(Paranthropus	P. Robustus
		(	P. Crassidens
		(Australo-pithecus	A. Africanus
		(	A. Transvaalensis

For the purpose, however, of choosing a starting point for modern man, it appears reasonable to say that man is distinguished from other related species by his culture and as is generally agreed man needed the ability to make tools, use fire and be able to communicate by oral methods to develop this culture. Consequently, the logical starting point for modern man would be the Pithecanthropus, as it was this hominid creature that first displayed these attributes. Although some disagreement exists as to exactly when Pithecanthropic creatures first appeared on the earth, the mean estimate (Brierley p. 38) appears to be 1,000,000 years ago, and for the purpose of this thesis this figure will be used.

Two important questions now need to be answered. How many were there, and what type of group situation did these hominid creatures live in?

Ansley Coale (p. 1) and Brierley (p. 34) agree that from a biological point of view there must have been a gene pool of some 10 to 100 thousand individuals for man to evolve to his present form. Coale suggest in fact that there well may have been more than 100,000 such individuals. If the fact is considered that these early ancestors were spread through Africa and Asia at least, then the starting figure of 100,000 individuals appears feasible. This, of course, is not intended to suggest that each of these individuals was at the same point of evolution, but rather that they formed many sub-species similar enough in characteristics to be called Pithecanthropic.

From considerations of the animal world it is unlikely that these hominids lived together in large groups. Normally, closely related species do not breed for if they do, the offspring is sterile. This preserves the close adaptation of the species to the environment in which they live. Hence, if large groups of Pithecanthropic creatures did live together in small areas, there seems to be no reason why these behavioural or territorial barriers would not come to be applicable in their case.

Saunders and Tweedie (p. 22) simulated the settlement of Polynesia by canoe loads of people and found that regardless of the starting population, once a population reached 50 it survived. As they point out, (p. 23) this is much lower than the number usually quoted in anthropological literature, but in their study they assumed a population would not become extinct if it reached 500 individuals and that no conclusions can be drawn as to the survival or extinction of population beyond this point. This simulation appears to closely approximate the type of evolutionary process our early ancestors were subject to.

Turnbull (p. 298) in a discussion of the Mbuti pygmies points out that the net hunters of this tribe live in relatively large bands of forty or fifty people, while many of the other specialist hunters and gatherers live in much smaller units, except for when the tribe meets periodically throughout the year. The culture of the Mbuti is more developed than would be expected of the Pithecanthropic hominids, but the size of groups needed to survive in a hunting and gathering situation is significant.

Lawrence (p. 112) in a study of the Australian aborigine suggests that although these people came together for cultural events, such as corroborees, they tended to spend most of their time in small groups except in areas where food was plentiful for much of the year.

From consideration of these and other similar findings, the hypothesis that the early hominids (Pithecanthropus) lived and worked in small groups to avoid the necessity for constant movement to new territories, yet lived in large enough groups to have reasonable chance of survival appears feasible. For the purpose of this thesis, a mean band size of 50 has been chosen.

CHAPTER II

HUNTING AND GATHERING ERA

(1,000,000 B.C. - 8,000 B. C.)

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HUNTING AND GATHERING ERA (1,000,000 B.C. - 8,000 B.C.)

Even with today's technology, man has achieved only a marginal victory over climatic change. The fuel shortage in North America during the winter of 1976 is just one example of how small this margin is. With this in mind, it is not difficult to realise the effects of the brutal climatic changes which occurred in the Pleistocene period, drastically altering the habits of prehistoric man, whose technology consisted solely of a primitive hammer. Table 2.1 shows the Ice Ages during the Pleistocene period and the corresponding developmental stages of man's culture.

Table 2.1*

GLACIAL STAGES	CIVILISATION	CULTURAL DIVISIONS
	METAL AGES	
POST GLACIAL	LATE PALEOLITHIC	MAGDALENIAN SALUTRIAN AURIGNACIAN PERIGORDIAN
WÜRM GLACIAL		
RISS WÜRM INTERGLACIAL	MIDDLE PALEOLITHIC	MOUSTERIAN (NEANDERTAL MAN)
RISS GLACIAL		ACHEULIAN
MINDEL-RISS INTERGLACIAL	EARLY PALEOLITHIC	
MINDEL GLACIAL		CHELLEAN
GUNZ-MINDEL INTERGLACIAL		
GUNZ GLACIAL		PRE-CHELLEAN
PREGLACIAL		

PALEOLITHIC

← WARM

COLD →

* (Stokes p. 400)

If the Antarctic continent is viewed as a monument to the last ice age, it is clear that any developing civilisation would be forced to migrate away from the advancing sheets of ice which characterised each of the Ice Ages. It has been calculated (Stokes p. 367) that sheets of ice radiated from the Scandinavian highlands covering 4.3 million square kilometres. Similar sheets of ice radiated out from the European Alps, centres in Northern Siberia, the Himalayas, North America, Antarctica, Hawaii, New Guinea, South America and Eastern Africa. Estimates indicate (Stokes p. 268) that an area of 39 million square kilometres of land surface was covered by ice during the last glacial stage. This represents some 27% of the total land area, and as it seems unlikely primitive man would choose to live at the edge of the ice sheets, then the percentage of habitable land area during this period was in the order of 70% or 100 million square kilometres.

The above figure of habitable land area does not consider the effects of the ice ages in areas the glaciers did not reach, and because of this is still too high. In such areas wet and dry periods are associated with glacial and interglacial periods and these also reduced habitable land area. During the wet periods many of the areas known today as tropical rain forest were uninhabitable, and during the dry periods lakes and rivers swollen with melting ice covered a large land area as did the oceans with their increased volume of water. When all of this is considered it seems reasonable that around 90 million square kilometres of land area were available for habitation during this period.

From Chapter 1 it is clear that at the beginning of this era man was developing in a very small portion of the world's land area and, as the period progressed, migrated slowly to other parts of the world. Thus to estimate population size around the middle of this period it is necessary to consider the migration pattern of developing man.

As Clark (p. 25) points out, it was only during the end of the Pleistocene period that man, due to his relatively unspecialised character and his culture, was able to spread his civilisation into the wider territories occupied today. It seems certain that our early ancestors were confined to the Old World. Clark (p. 25) summarises this area as follows: Africa - Algeria to the Cape, Europe - as far north as lowland England and central Germany, Western Asia - up to the mountains of northern Iran, India, South East Asia, Indonesia and as far east as the Makassar Strait.

The total land area of these combined regions is approximately 80 million square kilometres, but as a result of the climatic conditions mentioned above, a habitable land area figure would be much lower. For the purpose of this thesis a figure of 64 million square kilometres will be used. This represents 80% of total land area available rather than the overall average of 70% due to the fact that this area, as a whole, would be less affected by glacial and pluvial periods.

To estimate population size, the population density as well as land available for habitation needs to be known, and as Wilson (pp. 10-11) points out, population density varies due to type of society and, in the case of hunters and gatherers, the food producing capabilities of the land. A good idea of these population densities can be gained by considering the existing hunting and gathering societies.

Lawrence (p. 47) describes the climate of Central Australia as one where aridity prevails and the lack of surface water as a condition which limits the amount of flora and fauna available to a hunting and gathering society. He estimates (p. 72) that prior to the coming of the Europeans the Waljbiri tribe number some 1,000 persons and occupied an area of 98,000 square kilometres, giving a density of .01 persons/square kilometre. The Aranda tribe, also living in this area, have been estimated (Ibid p. 72) as having a population density of .03 persons/square kilometre. This latter figure is considered too high, however, by most sources, and a figure of .02 persons/square kilometre appears reasonable.

In contrast to this, the tribes living in the Riverina area of southeastern Australia, where climatic conditions are much less harsh and flora and fauna are available in large quantities, were much larger and occupied relatively less land area. The Ba:gundi, for example, has been estimated (Ibid p. 114) as having a population density of .08 persons/square kilometre. Even higher densities have been reported along the eastern coast of Australia near Port Jackson, and the generally accepted overall density of the Aboriginal population in Australia has been placed at .03 persons/square kilometre (Ibid p. 72)

Kroeber (pp. 389-390) estimates the hunting and gathering societies of ancient Mexico as having a population density of .03 persons/square kilometre, whilst the average density of other hunters and gatherers in North America was .02 persons/square kilometre, except for a narrow strip along the Pacific Coast from Alaska to California, where ideal conditions allowed a density of .2 persons/square kilometre.

Wilson (p. 11) suggests that although population densities of hunting and gathering societies may vary greatly, a figure of .03 persons/square kilometre is an accurate estimate of the average population density.

E. Deevey (p. 196) has estimated the population density to be .04 persons/square kilometre 25,000 years ago, and equates this with a population of 3.34 million. Both of these figures appear too high; the population density for reasons already stated, and the population size as it assumes that 84 million square kilometres of land was inhabited at this time, and as John Clarke (p.20) remarks, even less than 100 million square kilometres of land area is inhabited today.

Using a population density of .03 persons/square kilometre, and an area of 64 million square kilometres, the population circa 25,000 B.C. is calculated as about 1.9 million. This estimate will be used for this thesis.

The period from 1,000,000 B.C. to 8,000 B.C. was characterised by a very small growth rate, except at the end of the period when an increase in growth rate occurred as the clustering of population necessary for the emergence of an agricultural society began. The antecedent of the agricultural society would appear to be the development of a society generally known as primitive pastoralist and Wilson (p. 11) estimates a population density of 2 persons/square kilometre for this type of society.

However, the beginnings of agricultural society were very localised (see chapter III for more detail), and it is unlikely that this increase in population density affected much of the inhabited land area. The figure of 1 million square kilometres is appropriate for this purpose, and although it appears numerically large, it should be realised that this represents only 1% of the world's habitable land area, and in fact, about 1/40 of the land area of Africa. During this period it is unlikely that the population density of the other inhabited areas increased beyond the figure of .03 persons/square kilometre previously estimated.

Using population densities of 2/square kilometre for 1% of inhabited land area, and .03/square kilometre for the remaining 99% of land area, the population of 8000 B.C.can be calculated to be 5 million.

It is interesting to note that in the 17,000 years between 25,000 B.C. and 8,000 B.C. the increase in population was approximately 3 million, whilst the growth from 1,000,000 B.C. to 25,000 B.C. was only approximately 2 million. Thus, although the growth rate was still small by modern standards, the move toward agricultural society was beginning to herald an era of much larger population sizes.

CHAPTER III

AGRICULTURAL ERA

(8000 B.C. - 1650 A.D.)

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With the retreat of the last great ice cap around 25,000 B.C., large forests spread across much of Europe, Northern Asia and North America. The windswept interior plains became hard grass lands and the prairies south of the Mediterranean and southwest Asia became more desiccated and traversed by great rivers (U.N. Pop'n. Trends p. 5). But although the stage was set for the Neolithic "revolution", due to the hunting and gathering mentality of our ancestors, it was not to begin until around 8,000 B.C. and then only by accident rather than design in isolated communities. As Clark and Piggot (p. 157) suggest, a community whose economy is based on hunting and gathering is one of tactics rather than strategy, and of short-term rather than long-term decisions, whilst the agriculture based community, by virtue of the nature of the animal domestication and plant cultivation must inevitably be concerned with the long-term view. Consequently, the transition from one economy could not have been achieved easily or quickly.

There seems little doubt that agriculture had its beginnings in isolated pockets situated in the Near and Middle East as well as Southern and Southeast Asia (U.N. Pop'n. Trends p. 5), and that these pockets acted as foci for the spread of agriculture. However, evidence exists to show that whilst the skills of animal husbandry and agriculture were being learned, it was becoming common for relatively large groups of people to live together. As Clark and Piggot (p. 166) point out, an examination of the buildings at the Jericho III site suggest a population of some 2,000 people around

7,000 B.C. A similar analysis at Jarmo in Northern Iraq points to a village of population 150 at an even earlier stage. A complication here, however, is that these early farmers knew little of land regeneration, and it was necessary for them to move on after a number of generations to let the land lie fallow. Jarmo, for example, was estimated to be occupied for a period of 250 years, whilst a village of some 200 people discovered in the Rhineland and dated at 5,000 B.C . is suspected of being part of a cyclic chain of occupations. Tentative estimates have it as being occupied and re-occupied seven times in around 500 years, thus allowing ten years for each occupation and up to fifty years for regeneration of the soil (Ibid 169). Consequently care needs to be exercised in calculating the population size early in this period if multiple counting is to be avoided. In fact, it seems plausible that during the first two millenia of this era a clustering of existing population rather than a rapid increase of population was taking place.

However, by 4,000 B.C.(U.N. Popn. Bul.p.7) the first great urban civilization had appeared in Mesopotamia, Egypt, Crete and Western India. Also, much evidence has been found to prove that agricultural settlements had been established in Bulgaria, Yugoslavia, Central Europe, Russia and even as far afield as eastern France by this time. One such site, Trusesti, in Rumania, had a population estimated at 700 (Clark - Piggott p. 238), whilst the mean population of the other villages is estimated at 200 people. It is worth noting that at this point in time hunting and gathering populations still represented the existing culture in the New World, with the exception of a small area of what is now Mexico.

Edward Deevey (p. 197) has estimated the world population at this time as 86.5 million, with an assumed density of $1/Km^2$ in the Old World and $.04/Km^2$ in the New World. Whilst the assumed figure for the New World appears feasible, the density of $1/Km^2$ would appear too high. Wilson (p. 11) suggests that the maximum density for a primitive pastoralist society would be approximately 2.5/square kilometre and due to the variety of agricultural societies at this time in history this density will be used. However, much of the Old World culture was still hunting and gathering by nature and subject to a density of $.04/Km^2$. Consequently a density of $.6/Km^2$ will be used as the overall density of the Old World. The basis for this density is that in this era only one quarter of the Old World could truly be considered an agricultural based society, thus giving the .04 density figure a weight fact of three. On the basis of this density, the population at 4000 B.C. can be estimated at 47 million.

From 4000 B.C. to the birth of Christ the agricultural revolution at last became a revolution rather than an ominous rumble, and urban civilization developed in China, Central America, Peru and Central Europe. It was during this period that large population increases due to agriculture occurred. As Brierley points out (p. 77) by 2,000 B.C. the Sumerian city of Ur occupied some 150 acres and had a population reliably estimated at 24,000. Other Sumerian cities of Lagask, Umma and Khafajah have estimated populations of 19,000, 16,000 and 12,000 respectively at this time. Cities such as Memphis at the apex of the Nile delta in Egypt and Lo-Yang on the Yellow River in China had developed at this time and contained populations equivalent at least to those mentioned above. In India the brick built cities of Mohenjo-daro in the Indus valley and Harappa some 500 miles north were very large by this time, the ruins of the former covering at least a square mile and the latter having a walled perimeter of 2½ miles (Ibid p. 78). In the New World comparable development was taking place in the northern Andes and lower Mexico (Borrie p. 42) and although cities of large magnitude did not exist in central and northern Europe until a later date, there is a great amount of evidence (Cole p. 56) to suggest that many small towns of some 300 people did exist at this time.

Around the time of Christ (Borrie p.42) great trading nations with established industries as well as agriculture had been established and the Roman Empire had spread through much of Europe, Asia and Africa. The Han dynasty was well established in China by 0 A.D. and a census report put the population of China at 59.5 million (U.N. Pop'n. Trends p. 7). It should be noted that Usher (U.N. Pop'n. Trends p. 7) placed the population of China to be 71 million if the boundaries of 1920 China were considered. For this thesis the figure of 60 million will be used however, as this figure appears to be more widely accepted at this time. Much less extensive knowledge is available as to the population of India at this time. However, investigations by Professor Kingsley Davis, Pran Nath (Borrie p. 44) and Davis (U.N. Pop'n Trends p. 7) suggest a population of around 100 million at 0 A.D. This figure is supported to some extent by evidence which indicates one small kingdom having at least thirty seven towns, each with a population in excess of 5,000 inhabitants. Also the first recognised Indian empire under Chandragupta (c. 300 B.C.) left records indicating a standing army of some 700,000 men, the maintenance of which would have required a substantial population. J. Beloch's work of 1886 (Borrie p. 43) still stands as the authoritative work on the population of the Roman

Empire at the beginning of the Christian era. Although scholars have tended to raise these figures in certain areas due to rearrangement of boundaries, their basic validity has not been challenged. These estimates appear in Table 3.1

TABLE 3.1

POPULATION OF THE ROMAN EMPIRE (c. A.D. 14)

	NUMBER IN THOUSANDS	DENSITY/SQ. MILE
EUROPE		
Italy	6,000	62.2
Sicily	600	59.5
Sardinia-Corsica	500	39
Iberia	6,000	26
Narbonensis	1,500	39
Gaul	3,400	16.3
Danube	2,000	12.2
Greece	3,000	28.6
TOTAL	23,000	26
ASIA		
Asia (Province)	6,000	114
Asia Minor	7,000	44
Syria	6,000	143
Cyprus	500	135
TOTAL	19,500	77.5
AFRICA		
Egypt	5,000	465.0
Cyrenaica	500	85.5
Province of Africa	6,000	39.0
TOTAL	11,500	67.5
ROMAN EMPIRE	54,000	41.5

If the estimates at the beginning of the Christian era for China (60 million), India (100 million) and Roman Empire (54 million) are totalled, a figure of 214 million is obtained for the world's population. However, this estimate needs to be raised to include the population of Africa south of the Sahara, south-eastern Asia, Northern Europe and the Americas. The consensus of opinion (U.N. Pop'n. Trends p. 8), (Borrie p. 44) puts the population of these areas between 75 and 85 million. For the purposes of this thesis the figure of 85 million will be used, giving a world population of 300 million at the beginning of the Christian era. Coale (p. 17) suggests 200 million as the population at this time, as does Borrie (p. 44) and the only radical departure from this figure appears to be the 133 million suggested by E. J. Deevey (p. 197) which has not been supported by other sources to date.

Thus, after many thousands of years of virtually no growth the population leapt from the 5 million of 8000 B.C.to 300 million by 0 A.D., but in doing so had created a set of circumstances that would once again slow the rate of population growth. With the growth of the great cities came the plagues which lowered the life expectancy by many years. Russell, as quoted in Borrie (p. 52) puts the life expectancy in Rome at 15.3 years, and a later work by Durand (Borrie p. 53) supports this figure by estimating a life expectancy of 20-30 years in the first centuries of the Christian era. Clark (p. 64) using a population figure of 256 million for 0 A.D. suggests that by 1,000 A.D. the population had only risen by some 24 million. Another contributing factor to this lower rate of growh was the decline of the Roman Empire and the ensuing wars which was an occurrence not without parallel in China and India. In China, with the fall of the Han dynasty, a long period of disintegration followed, with the population being ravaged by warfare until order was re-established under the Sung Dynasty. To obtain an idea of the effects of these factors on population growth the specific case of Britain can be considered. Monastic records in Britain (U.N. Pop'n. Trends p. 9) show a population of 3.7 million in 1348, and following the outbreak of bubonic plague a population of some 2.1 million 30 years later! However, whilst very small increases or even decreases were taking place in the ancient centres of dense population, the "frontier" regions were being subjected to the rapid increases in population that had taken place earlier in the more settled regions. The steady population growth that took place between 0 A.D. and 1650 can be summarised into three distinct areas. The following summary is from the Determinants and Consequences of Population Trends (U.N. 1953 p. 10)

1) Wide fluctuations, with relatively small net increase, and in some case net decrease, in ancient centres of dense population: China, India, Mesopotamia, the Near East and Egypt.

2) Similar but less violent fluctuations with an emergent trend toward population increase, in southern and western Europe - with marked decreases at different times in Greece, Spain and the Danubian region.

3) Pronounced increase in the "frontier" regions of central and eastern Europe.

Several estimates of the population size in 1650 have been made, the most widely quoted of these being the estimates of Carr-Saunders and Willcox, and seem only to differ in their analysis of the population of China at this time. The Ming system of population records ran from 1381 to 1620 and suggest a population of 52 million at the beginning of the seventeenth century. However, it needs to be remembered that the purpose of these statistics was tax collection, and evasion and bribery must have influenced them greatly. Durand and Ping-ti H. (Borrie p. 51) independently came to the conclusion that these figures were virtually useless by the fifteenth century and that a population of 150 million in 1650 would be more realistic than the proposed 52 million. Carr-Saunders arrived at this figure also by working back from eighteenth century records which are generally considered more reliable than the Ming statistics. Willcox, who had previously accepted the figure of 52 million, has also revised this figure to 113 million (Borrie p. 51). Thus, a figure of 545 million (Carr-Saunders estimate) will be used as the world population in 1650. The details of this estimate appear in the table 3.2. It is interesting to note that Clarke (p. 64) postulates a world population of some 516 million in 1650, but with an unrevised figure of 100 million as the population of China. This would seem to add further credence to the figure of 545 million.

TABLE 3.2

ESTIMATION OF WORLD POPULATION 1650 (millions)

Africa	100
North America	1
Latin America	12
Asia (exc. U.S.S.R.)	327
Europe and Asiatic U.S.S.R.	103
Oceania	2
WORLD TOTAL	545

CHAPTER IV

THE INDUSTRIAL ERA

(1650 - 1976)

CHAPTER IV

THE INDUSTRIAL ERA (1650 - 1976)

It is not surprising that this era is considered one of great population explosion when it is noted that in 300 years the population has grown to some eight times the number it had taken almost a million years to reach in 1650. This is an era of staggering technological advancement, great migrations and a rapidly decreasing death rate leading to a much higher average life expectancy. The timing and intensity of these changes varies greatly in different areas of the world, and consequently these areas will be considered separately. For this era the population estimates of Carr-Saunders will be used for the period 1650-1900 and those of the United Nations for the period 1900-1976. These estimates appear in Table 4.1 and average annual rates of growth in selected areas appear in Table 4.2.

TABLE 4.1

ESTIMATES OF WORLD POPULATION BY REGION 1650-1950

ESTIMATED POPULATION IN MILLIONS

	WORLD	AFRICA	N.AMERICA	L.AMERICA	ASIA (EXC.USSR)	EUROPE USSR	OCEANIA
Clarke-Saunders Est.							
1650	545	100	1	12	327	103	2
1750	728	95	1	11	475	144	2
1800	906	90	6	19	597	192	2
1850	1,171	95	26	33	741	274	2
1900	1,608	120	81	63	915	423	6
U.N. Est.							
1920	1,834	136	115	92	997	485	9
1930	2,008	155	134	110	1,069	530	10
1940	2,216	177	144	132	1,173	579	11
1950	2,406	199	166	162	1,272	594	13

The decreasing population in Africa between 1650 and 1850 represents the effect of the slave trade. The fact that increases in population, particularly in North America, do not show a correspondingly large increase is not surprising, as it has been estimated (U.N. Pop'n. Trends p. 12) that at least one quarter of the slaves removed from Africa died in transit to the New World. This large unwilling migration does need to be considered, however, when analysing the growth rates in Table 4.2.

TABLE 4.2

AVERAGE ANNUAL RATES OF INCREASE IN ESTIMATED POPULATION OF THE WORLD AND THE AREA OF EUROPEAN SETTLEMENTS, 1650-1950

	Annual Increse per thousand	
Period	World	Europe, Asiatic U.S.S.R., America Oceania
1650-1950	5	7
1650-1750	3	3
1750-1800	4	7
1800-1850	5	9
1850-1900	6	11
1900-1950	8	10
1900-1920	7	10
1920-1930	9	11
1930-1940	10	10
1940-1950	8	8

From this table (4.2) it can be seen that for the entire period 1650-1950 the annual rates of population growth of European settlements have been consistently above the world average. It is commonly argued that this is a result of the rapid strides made in medical science during the period, but this is an oversimplification. In fact, no large gains in life expectancy were made in the European settlements until the late nineteenth century (U.N. Pop'n. Trends p. 54). Rather, much of the accelerated population growth in these areas can be explained by the improving efficiency of agricultural and industrial technologies which helped provide an economic climate in which marriages occurred earlier and more frequently than before (Borrie. p. 67). As a direct result of this the birth rate increased rapidly during this period. This phenomena is not without precedent in modern times. When the post World War II economies of U.S.A., Australia, New Zealand and many Western European countries provided rising wages and full employment a sharp rise in marriage and birth rates occurred (Ibid p. 67). This increasing birth rate does not, however, explain the rapid rise in population unless a decrease in death rate occurred. This, in fact, appears to be the case, although evidence suggests it happened by accident rather than design. The plague, for example, occurred in England for the last time in 1666, and as Borrie (p. 66) points out, this was probably due to an "obscure revolution in the animal kingdom" which replaced the plague carrying black rat by the relatively safe brown rat.

This type of rapid growth was not to last, however, because the unsanitary conditions prevailing in the developing industrial metropolises soon produced new killer diseases. Typhus and cholera went hand in hand with the cities at this time and average life expectancies of as low as 25 years were common. Smallpox also was rife in European settlements at this time and was an added factor in the increasing death rate.

By 1850, general improvements in standard of living had occurred, and although crowded housing was still common, much improved public health facilities as well as better food and medical knowledge once again led to a falling death rate and from this time to the present any reduction in growth rate has been due to a conscious effort to control the birth rate. Birth control is usually thought of as a modern occurrence, but there is evidence to suggest (Borrie p. 70) that it was being practised in European settlements midway through the nineteenth century. This may well explain that although a sustained period of growth did take place in nineteenth century Europe, the growth rate did not even approach the rate estimated for many nations in the twentieth century.

The great stride in medical knowledge at the beginning of the twentieth century in European settlements caused the death rate to plummet, and even though birth control was being practised in these areas, the real population explosion began and it is obvious that this can only be controlled if all segments of the community consciously attempt to lower the birth rate. In the one hundred years from 1840 to 1940 the average life expectancy in European settlements rose a staggering 23 years and to the present day by an even larger margin (U.N. Pop'n. Trends p. 54)

Thus the population growth of the European settlements from 1650-1976 can be summarised into the following three categories.

1) 1650-1850 - Population growth due to increasing birth rate and stable or slightly decreasing death rate.

2) 1850-1900 - Increasing death rate early in the period, but for most of the time a decreasing death rate due to better organisation of heavily populated areas. During this period the birth rate gradually began to fall.

3) 1850-1976 - Rapidly decreasing death rate due to the development of many new medical techniques. A falling birth rate due to attempts at birth control, but during the twentieth century this decrease has not even approximated the decrease in death rate.

The development in the non European settlements has been characterised by high birth rates and correspondingly high death rates until the last thirty or so years. Comparisons of birth and death rates for European and non European settlements are given in Tables 4.2 and 4.4. These tables are summarised from "The Determinants and Consequences of Population Trends" (U.N. 1953).

TABLE 4.3

DEATH PER 1,000 POPULATION SELECTED COUNTRIES 1932

Egypt	28.5
British Guiana	21.2
Chile	22.7
Mexico	26.1
Puerto Rica	22.0
Ceylon	20.5
Japan	17.7
Austria	12.4
Spain	13.1
Italy	12.0
Norway	9.1
Poland	12.5
Greece	12.6

Note:

As death rates in countries with high mortality are generally considered to be given as lower than their actual value, the lowest provincial rate for European settlements has been used in order to make the comparison more realistic.

TABLE 4.4

BIRTH RATES FOR REGIONS OF THE WORLD (c. 1947)

Region	Births/1000
Africa	40-45
U.S. and Canada	25
Latin America	40
Near East	40-45
South-central Asia	40-45
Japan	31
Remaining Far East	40-45
North-west-central Europe	19
Southern Europe	23
Eastern Europe	28
Oceania	28

It is clear from an examination of these tables that the pattern of growth in the non European settlements had changed very little prior to 1920 from what it was during the preceding millenia. Since 1920 many of these countries have undergone the "industrial" and "medical" revolution, and the brutally high death rates have fallen rapidly as happened in the European settlements some one hundred years previously. Unlike the European settlements, however, the birth rate has not undergone a comparable decrease. In fact, if anything the birth rate has tended to increase and the populations of these countries have skyrocketed in a truly Malthusian sense. Africa, for example, took almost 300 years for its population to rise from 100 to 136 million, yet from 1920 to 1950 the population rose from 136 to 199 million (U.N. Pop'n. Trends p. 11). Similar patterns of growth can be seen in Latin America, India and Asia.

This type of growth approximates the growth of the European settlements in the seventeenth and early eighteenth centuries, but it seems unlikely that it will be checked by outbreaks of new diseases as was the case with the Europeans. Also due to religion and culture in many of the non European settlements it seems unlikely that the population will make any serious attempt at controlling the birth rate. The fall of the Ghandi government in India in 1977 has partially been explained by the rejection of the populace of birth control measures advocated and, secretly in some cases, practised, is just one example of the unlikely acceptance of birth control in some areas.

Many projections as to the likely outcome of the current population explosion have been made, but at the present moment all that seems certain is that the population will continue to grow very rapidly in the countries of non European settlement and as the death rate is reduced further this growth rate will increase.

SUMMARY

For most of man's history, the growth rate has been very small and for some thousands of years the probability of man surviving to inhabit the Earth as he does today was even smaller. With the end of the last Ice Age climatic changes enabled man to spread his sphere of influence and the wandering of his hunting and gathering ancestors enabled the conditions for agricultural development to appear. This primitive, at first, form of agriculture spread from Africa and Asia into Europe and the first large population centres came into existence. Ironically, perhaps, the next great stage in man's development; the Industrial Revolution, began in Europe and slowly spread, and is spreading, to the countries from which the Agricultural Revolution came. With the Industrial Revolution came the first great periods of growth and this has since been compounded with advances in medical science to give even greater growth rates.

In most of the European settlements this large growth rate has been partially checked by a decreasing birth rate, but at the moment many of the other countries have populations which are increasing at an alarming rate and it appears this will not change in the foreseeable future. One thing is certain, and that is unless this growth rate is checked then Malthus will win an eleventh hour victory over Ricardo.

The data to be used in this thesis appears in Table 4.5. As parallel development did not occur from 1650 onwards, the world population is quoted in areas for this period. These estimates are only a guide to the endpoints of each period and a more detailed analysis of each period appears in later chapters.

TABLE 4.5

WORLD POPULATION (IN MILLIONS) 1,000,000 B.C. - 1976

YEAR	WORLD	EUROPEAN[1]	NON EUROPEAN
1976	4,000	-	-
1940	2,216	866	1,350
1900	1,608	573	1,035
1850	1,171	335	836
1800	906	219	687
1750	728	158	570
1650	545	118	427
0 A.D.	300	-	-
4000 B.C.	47	-	-
8000 B.C.	5	-	-
25,000 B.C.	1.9	-	-
300,000 B.C.	-	-	-
1,000,000 B.C.	0.1	-	-

1 - Includes North and Latin America, Europe and Asiatic U.S.S.R., and Oceania.

CHAPTER V

CUMULATIVE POPULATION 1,000,000 B.C. - 8,000 B.C.

CHAPTER V

1,000,000 B.C. - 8,000 B.C.

Using a starting population of 100,000 and a mean band size of 50 suggests that the population of this era can be considered as consisting of 2,000 bands of individuals spread widely across the Old World. As this time period progressed many of these bands would become extinct due to such events as wars or famine, while others would grow much larger than the original 50. It is also quite possible, due to evolution, that some of these original bands developed into species other than early man. These occurrences notwithstanding, migration of individuals from one group to another must have occurred from time to time. In fact, as the number of groups diminished, leaving only the most successful, migration would have tended to increase as flourishing societies are more capable, and hence more willing to accept immigrants.

If these factors are considered then it suggests that the population growth in this period can be simulated by a statistical model. Furthermore, Kendall's birth, death and migration model (Pollard p. 65) appears particularly applicable to this problem and it is used below, modified only to allow for a starting population of N_o rather than 0 individuals (See Appendix A).

The modified model yields the following probability generating function:

$$\phi(z,t) = \left\{\frac{\lambda-\mu}{\lambda z-\mu-\lambda(z-1)e^{(\lambda-\mu)t}}\right\}^{\nu/\lambda} \left\{\frac{\lambda z-\mu-\mu(z-1)e^{(\lambda-\mu)t}}{\lambda z-\mu-\lambda(z-1)e^{(\lambda-\mu)t}}\right\}^{N_o} \quad \{V1\}$$

where λ is the birth rate, μ the death rate, ν the immigration rate and N_o the starting population.

Clearly, extinction cannot occur in this model, as even though the population may become zero, there is always the probability of an immigrant in each time interval

The probability of there being zero persons at time t is given by the relationship:

$$\phi(0,t) = \left\{\frac{\lambda-\mu}{\lambda e^{(\lambda-\mu)t}-\mu}\right\}^{\nu/\lambda} \left\{\frac{\mu e^{(\lambda-\mu)t}-\mu}{\lambda e^{(\lambda-\mu)t}-\mu}\right\}^{N_o} \qquad \{V2\}$$

The mean number of persons in the population at time t is given by:

$$M(t) = \frac{\nu}{\lambda-\mu}\{e^{(\lambda-\mu)t}-1\} + N_o\, e^{(\lambda-\mu)t} \qquad \{V3\}$$

During this period the average life expectancy was 20 years (see for example Coale p. 18) and this automatically fixes the birth rate at 5 per thousand (see Appendix B). A migration rate of .5 per thousand is used and this appears feasible, particularly in the light of evidence presented by Davis pp. 53-65. As noted previously, the initial population to be used is 50 individuals.

Assuming a steady growth rate throughout the period, the population can be described by

$$N(t) = N_o e^{kt} \quad \text{where } k = (\lambda - \mu) \text{ is the average growth rate.}$$

In this case

$$5 \times 10^6 = 10^5 \, e^{10^6 k}$$

giving $k = 3.9 \times 10^{-6}$

hence $\mu = -k + \lambda$

$= .04999$

Now using the above data, from {V2} the probability of a group having zero population at time t is:

$$\phi(0,t) = \left\{\frac{3.9 \times 10^{-6}}{.05e^{3.9\text{x}10^{-6}t} - .04999}\right\}^{.1} \left\{\frac{.04999e^{3.9\text{x}10^{-6}t} - .04999}{.05e^{3.9\text{x}10^{-6}t} - .04999}\right\}^{50}$$

$$\text{and } M(t) = 1282.05 \{e^{3.9\text{x}10^{-6}t} - 1\} + 50\, e^{3.9\text{x}10^{-6}t}$$

The total population based on a starting figure of 2,000 groups can then be calculated by

$$N = \{2{,}000 - 2{,}000\, \phi(0,t)\} \;\; M(t)$$

TABLE 5.1

TIME	GROUPS SURVIVING	MEAN POP'LN.	TOTAL POP'LN.
0	2000	50	100,000
1,000	1502	56	84,112
2,000	1336	58	77,488
3,000	1238	61	75,518
4,000	1138	67	76,246
5,000	1090	72	78,480
6,000	1064	78	82,992
7,000	1050	84	88,200
8,000	1044	90	93,960
9,000	1040	96	99,840
10,000	1038	102	105,876

Table 5.1 illustrates population behaviour for t=0 to t = 10,000 using the above model. The number of surviving groups has been rounded down to an integer and total population therefore is slightly low in most cases.

The total population over this 10,000 year period oscillates around the starting population of 100,000 individuals, however, the number of groups has been almost halved and the population of those surviving doubled. During this period there is little evidence to suggest that any major changes in life style occurred and consequently the mean band size of 102 persons is too large for the group to exist as one unit. More likely is the situation existing in Australian aboriginal society where the tribe (or main group) is split into smaller groups for all but special occasions. In all probability, then, the situation after 10,000 years could be viewed as being almost identical to that of the beginning. In fact, there appears little reason for this situation to change greatly until the necessary changes in society occurred. From a historical point of view little change occurred in the first 900,000 years of this period (see Chapter II) and consequently a stationary population of 100,000 individuals will be used for this period. It is important to note that at various times there would have been many more than 100,000 hominid creatures alive, but a large number of these would represent evolutionary strains long since vanished and any affect they may have had on our ancestors is provided for in the model by allowing immigration to occur.

Assuming the above, the population grew from 100,000 to 5,000,000 during the last 100,000 years of this period and further from the population estimates (Chapter II) it can be seen that an increase of 3,000,000 occurred in the last 25,000 before 8,000 B.C. This type of growth pattern could be expected as our ancestors were moving toward a primitive pastoralist society at this time and the advantages of such a society would make a miniature population explosion likely. Also the advantages of a better organised society would tend to lower the death rate and the increase in population would resemble an exponential growth curve. This type of situation parallels somewhat the exponential nature of population growth in non-European countries at the present time although the modern period has many factors which make a simple exponential growth curve impractical (see Chapter VII).

For the period 108,000 B.C. - 8,000 B.C. the following model is used to represent population growth:

$$N(t) = N_o e^{kt} \qquad \text{where } N(t) = N_o \text{ when } t = 0$$

In particular for $t = 0$, $N_o = .1 \times 10^6$ and $t = 100{,}000$, $N = 5 \times 10^6$ the explicit relationship is given by:

$$N(t) = .1 \times 10^6 \, e^{3.95 \times 10^{-5} t}$$

Table 5.2 shows the predicted values as compared to the estimated values for particular periods of time.

TABLE 5.2

TIME	POPULATION (in millions)	
	DATA	MODEL
108,000 B.C.	.1	.1
33,000 B.C.	1.9	1.93
8,000 B.C.	5	5.1

Therefore the population of this period can be viewed as having been stationary for the first 900,000 years at 100,000 and then growing exponentially to reach 5 million by 8,000 B.C.

FIGURE 5.1

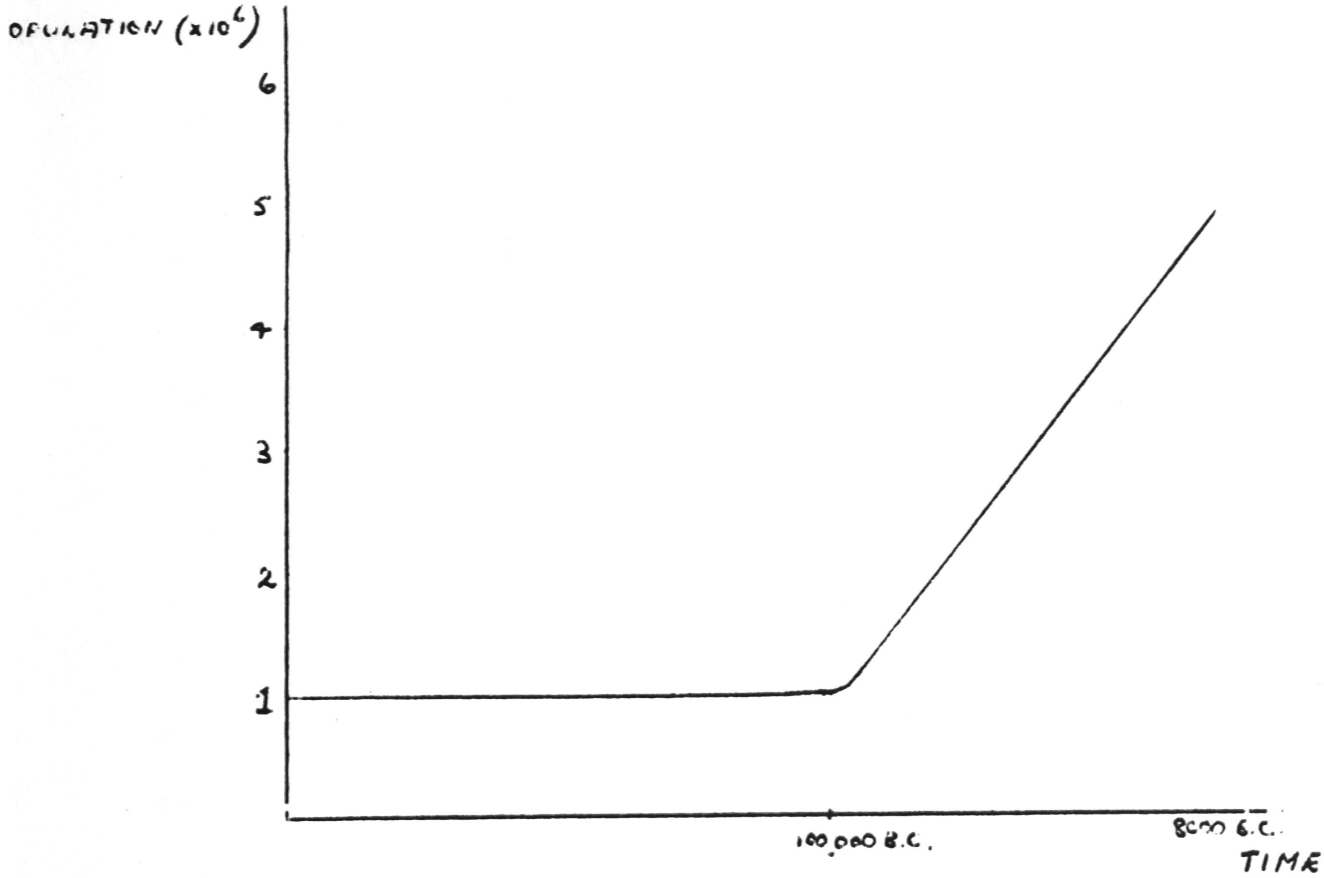


Figure 5.1 illustrates the growth pattern for this period. Due to the long period of time involved the horizontal axis is not to scale.

The cumulative population of this period, assuming an average life expectancy of 20 years, can then be calculated by the relationship:

$$C.P. \quad = \quad \frac{1}{20}\ (100{,}000 \times 900{,}000) + \frac{100{,}000}{20} \int_{0}^{100{,}000} e^{3.95\text{x}10^{-5t}}\, dt$$

$$= \quad 11 \times 10^{9}$$

Therefore the cumulative population for this period is 11 billion. This figure may appear small when the length of the time period is considered. However, it should be noted that only ancestors of modern man rather than all hominid creatures are being counted. Also the remains of these societies found to date are very localised until late in the period which suggests that our ancestors were very small in numbers for a great many years. Deevey (p. 197) estimates 36 billion for this period and in defence of this figure points out how relatively common stone tools are as fossils. While this is true for the end of the period it is not the case for the beginning and middle portions, and thus a reduction to a little less than one third of this figure does not appear outrageous. The models of Keyfitz (p. 581, 1966) and Wellemeyer (p. 18) estimate populations of 13×10^9 and 12×10^9 respectively and this adds further credence to the model developed in this chapter.

CHAPTER VI

CUMULATIVE POPULATION 8000 B.C. - 1650 A.D.

CUMULATIVE POPULATION 8000 B.C. - 1650 A.D.

The population growth for this period is usually described by an exponential relationship as can be seen from the models of Keyfitz (1966, p. 581), Wellemeyer (p. 18) and Deevey (p. 195). It should also be noted that Winkler's model (p. 73) is based on the compound interest formula which is concerned with discreet data and is analagous to the exponential model when continuous data is considered. The exponential model, however, proves unsatisfactory in describing the population growth of this era and it is instructive to look at the major reasons for this before proceeding to a discussion of the model to be used in this thesis.

The exponential model is stated explicitly as:

$$N(t) = N_o e^{kt} \qquad (1)$$

where N(t) is the population at time t, N_o is the initial population and k is the average growth rate.

For this period $N_o = 5 \times 10^6$ and $N(t) = 545 \times 10^6$ when $t = 9650$ giving $k = 4.8 \times 10^{-4}$

Thus the relationship can be stated explicitly as:

$$N(t) = 5 \times 10^6 \, e^{4.8 \times 10^{-4}t} \qquad (2)$$

Table 6.1 shows the population for selected years in this period using the above relationship and compares these figures to those estimated in earlier chapters.

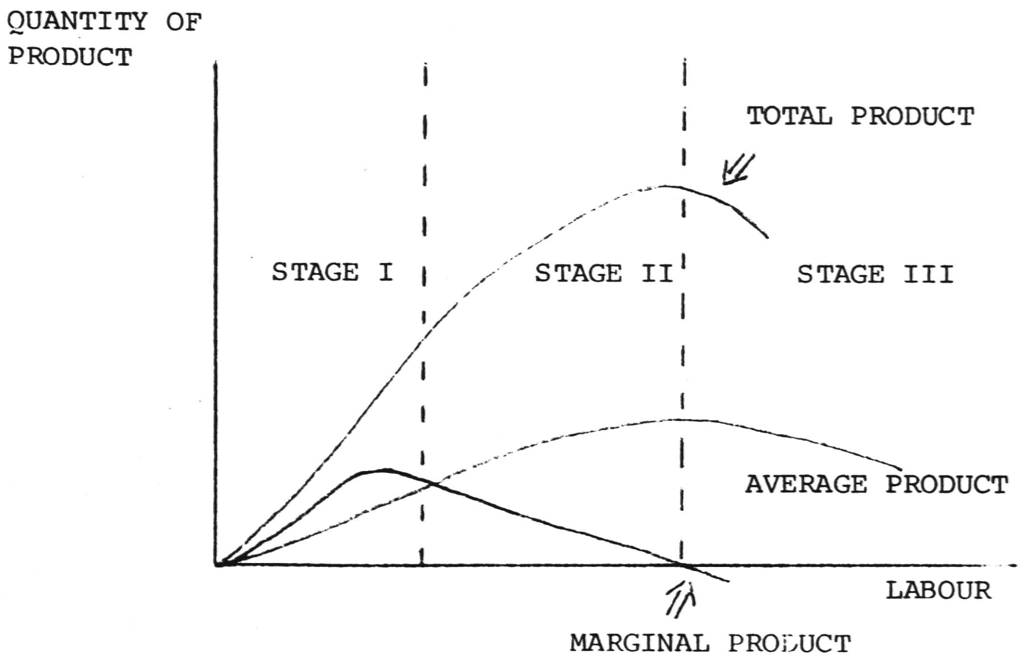
TABLE 6.1

Year	Population (millions)	
	Data	Model
8000 B.C.	5	5
4000 B.C.	47	34
0 A.D.	300	233
1650 A.D.	545	545

The predicted populations of 34 and 233 million are both too low in the light of the evidence, presented in Chapter III, which suggested that the estimates of 47 and 200 million would, if in error, tend to be conservative. There are, however, even more important anomalies in the exponential model. In particular, the study of the economics of agricultural society depends heavily upon the Law of Diminishing Marginal Returns (Mansfield pp. 122 - 126) and as the population growth of this period depended upon the emergence of agricultural society, a connection between the two appears plausible, if not, in fact, obligatory. If this connection is accepted then the following discussion not only shows the unsuitability of the exponential model but suggests the general characteristics that a realistic model should possess. A full analysis of the Law of Diminishing Marginal Returns can be found in Mansfield pp. 126 - 129 and only the main results are summarised here. In agricultural society the variable input is viewed as labour and is considered as such in the following analysis.

Figure VI - I shows the various stages of production along with the relevant average and marginal product curves.

FIGURE VI - I



If total product is considered as a function of amount of labour, i.e. T.P. = f(L), then:

$$\text{marginal product (M.P.)} = \frac{d}{dL}(f(L))$$

$$\text{and} \quad \text{average product (A.P.)} = \frac{f(L)}{L}$$

Throughout Stage I the average product is increasing which implies that the addition of more labour will increase the average amount each person receives. Also, the marginal product reaches its maximum in this stage which indicates that rate of increase of average product will reach a maximum in this stage and then begin to fall. In Stage II the marginal product decreases more rapidly and average product reaches its maximum which means that from this point on the addition of more labour will lower the average product even though the total product continues to increase. Under normal circumstances, businesses operate in this stage. After this (Stage III) the marginal product becomes negative, showing that with each additional unit of labour not only does average product decrease but also total product.

The implications of this analysis are far reaching when purely agricultural society is considered.

While the developing agricultural society was in the Stage I situation it is clear that rapid population growth would be desirable, as the additional labour available would increase the average product thus giving all a larger share of the agricultural pie. Also, as marginal product is increasing to a maximum in this stage, rapid population growth could be expected. This is comparable to the post-World War II economics, discussed earlier, where rapid population growth is clearly documented. Stage II would suggest a continuing increase in population but as the average product began to fall the rate of population increase would fall also. In Stage III the average product begins to fall rapidly and approaches zero. Clearly the population would react, perhaps involuntarily, before this happened and the growth rate would become commensurate with that required to maintain the maximum feasible population.

With this in mind the exponential model is unsatisfactory and should be replaced by a model proposing an "S" or sigmoid growth curve. The need for a maximum population size and a sigmoid shape suggests a model based on the logistic equation used by Pearl and Reid in 1920 to simulate population growth in the United States. This particular model has been criticised in recent years as its projections have been shown to be inaccurate. It should be remembered, however, that the making of predictions for the future from any trend model can at best be described as an elaborate form of guesswork. As this thesis is concerned with past trends rather than predictions, the logistic model is viewed as being applicable.

The logistic relationship is stated explicitly as:

$$\frac{dN}{dt} = kN(C-N) \qquad (3)$$

where $N = N(t)$ is the population at time t with k and C constants. From a consideration of $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$ it is clear that N(t) is a maximum when $N = C$ thus showing C as the carrying capacity of the population for the given inputs. Also from (3), the maximum rate of population growth occurs when $N(t) = \frac{C}{2}$.

The above differential equation (3), can be solved by separation of variables to yield:

$$N(t) = \frac{C}{1+(\frac{C}{N_o}-1)e^{-kct}} \quad \text{where } N(t) = N_o \text{ when } t = 0 \quad (4)$$

From (4) $$-kC = \ln\left(\frac{\frac{C}{N}-1}{\frac{C}{N_o}-1}\right)/t \qquad (5)$$

For this particular model using the inputs t = 0, 8000, 9650 for N = 5, 300 and 545 the following simultaneous equations occur.

$$-kc = \mathrm{Ln}\left(\frac{\frac{C}{545} - 1}{\frac{C}{5} - 1}\right) / \ 9650$$

$$-kc = \mathrm{Ln}\left(\frac{\frac{C}{300} - 1}{\frac{C}{5} - 1}\right) / \ 8000$$

These two equations can be simplified to give:

$$\left(\frac{\frac{C}{300} - 1}{\frac{C}{5} - 1}\right)^{1.2} - \left(\frac{\frac{C}{545} - 1}{\frac{C}{5} \quad 1 \quad 1}\right) = 0$$

which is easily solved to show C = 1328. This value can then be substituted into (5) giving kC = .00054.

For this particular model (4) can be written emplicitly as

$$N(t) = \frac{1328}{1 + 264.6e^{-.00054t}}$$

Table 6.2 compares estimated population size with population size specified by the above model.

TABLE 6.2

Year	Population (millions)	
	Data	Model
8000 B.C.	5	5
4000 B.C.	47	43
0 A.D.	300	295
1650 A.D.	545	543

Table 6.2 illustrates the "closeness of fit" of the data to the predicted values. The value of 43 million predicted for 4000 B.C. is somewhat low but well within the acceptable range of error when the level of accuracy of the data is considered. Furthermore, the maximum rate of population growth of the model occurs when the population reaches 664 million and this agrees with the rapid population growth documented around 1650, and the fact that modern society with all its technological developments has reached a crisis with the population at 4 billion suggests that a maximum population size of 1328 million for purely agricultural society would be feasible.

As a final remark concerning the suitability of the logistic equation it is necessary to realise that the total product curve discussed earlier is formed by amalgamating many such curves dealing with smaller areas. Thus, the logistic equation used here is an amalgamation of many logistic curves relating to different areas. In this way the population development, which was by no means parallel, in the various areas is accounted for.

In the relationship:

$$\frac{dN}{dt} = KN(C - N)$$

KNC can be considered as the birth rate and KN^2 the death rate. It is clear that for small N the birth rate is considerably larger than the death rate and this explains the population explosion in each area when the transition to the particular logistic equation for that area occurred.

The average life expectancy during this period can have been little different to the 20 year life expectancy of the preceding period. Russell (Borrie p. 52) puts the life expectancy in Rome at 15.3 years and later work by Durand (Ibid p. 52) suggests an average age between 20 and 30 years. If the plagues and associated problems of the developing agricultural metropolises are considered the lower estimate of 20 years appears more realistic. Also, during this period little had occurred to remedy the very high infant mortality rate which tends to lower the average life expectancy to a figure which is much less than the age a person could expect to attain should they survive infanthood.

Using an average life expectancy of 20 years the cumulative population (in millions) for this period can be found by

$$C.P. = \frac{1328}{20} \int_{0}^{9650} \frac{dt}{1 + 254.6e^{-.00054t}}$$

$$= 66.4 \left(\frac{1}{.00054} \text{ Ln } (e^{.00054t} + 254.6) \Big|_{0}^{9650} \right)$$

$$= 64,252$$

Therefore the cumulative population for the period 8000 B.C.-1650 A.D. is calculated as 64.3 billion.

It is interesting to note that for the first 1 million years the cumulative population was less than one quarter of that of the succeeding 10,000 years and should this trend continue it would have serious implications for a world of finite resources.

CHAPTER VII

CUMULATIVE POPULATION 1650 - 1976

CUMULATIVE POPULATION 1650 - 1976

During this period, ironically the shortest to date, the human population has grown at a rate completely unprecedented in the history of man. It is usual for this growth to be described as exponential (see for example the models mentioned at the beginning of Chapter VI), but as the following brief analysis of the exponential model illustrates, the word is used in the colloquial rather than the mathematical sense.

The exponential model can be represented as:

$$N(t) = N_o e^{kt}$$

where $N(t)$ is the population at time t, N_o the population at time $t = 0$ and k the growth rate.

Letting $N_o = 545 \times 10^6$ and $N(t) = 4 \times 10^9$ when $t = 325$ gives $k = .0061$.

The relationship can then be stated explicitly as:

$$N(t) = 545e^{.0061t} \qquad (\ N(t) \text{ in millions }\ )$$

Table 7.1 compares estimated population with predicted population size for this particular model. It should be noted that different boundary conditions will yield different values for k, however Table 7.1 is representative of the "goodness of fit" of these other models.

TABLE 7.1

Year	Population (in millions)	
	Data	Model
1650	545	545
1750	728	1003
1800	906	1361
1850	1171	1846
1900	1608	2504
1950	2400	3397
1976	4000	4000

It is clear from Table 7.1 that the model does not accurately simulate the actual population growth. Furthermore, a study of the data in Table 7.1 points out that the population of 1650 took over 200 years to double, while the population of 1900 almost trebled in 76 years which is incompatible with the constant doubling time $(\frac{Ln2}{k})$ of the exponential model. Finally the exponential model places no upper bound on the population size and this appears most unrealistic in the light of the current economic and political climate.

The requirement of a maximum value suggests the use of the logistic model. However, it is also unsatisfactory for the following reasons.

Consider the following relationship:

$$N = \frac{C}{1 + (\frac{C}{N_o} - 1)e^{-kt}}$$

Let D be the amount of time required at time t in order that the population double.

Then
$$\frac{N(t + D)}{N(t)} = 2$$

$$\frac{1 + (\frac{C}{N_o} - 1)e^{-kt}}{1 + (\frac{C}{N_o} - 1)e^{-k(t + D)}} = 2 \qquad k > 0$$

giving
$$e^{kD} = \frac{2(C - N_o)}{C - N_o - N_o e^{kt}}$$

$$D = \left[\ln 2(C-N_O) - \ln (C-N_O-N_Oe^{kt})\right] /k$$

Clearly as t becomes larger the doubling time increases and this is in direct contrast to the trend in the observed data. Furthermore, if t_0, t_1, t_2 are three equally spaced time periods with population size N_o, N_1, N_2 respectively, then the maximum population value (c) for the logistic model is given by:

$$C = \frac{2x_1 - (x_0 + x_2)}{x_1^2 - x_0x_2} \qquad \text{where } x = \frac{1}{N}$$

In particular if $t_0 = 0$, $t_1 = 150$, $t_2 = 300$, giving $N_0 = 545 \times 10^6$, $N_1 = 906 \times 10^6$ and $N_2 = 2400 \times 10^6$ (i.e. to correspond to 1650 A.D.) then

$$C = \frac{.0022 - (.0018 + 00042)}{.0000012 - .00000076}$$

$$= -45.5$$

Obviously, this is not the maximum population value and illustrates that the data does not occur on the lower shaped portion of the logistic curve which is the only portion applicable to the growth of human population.

Neither the exponential nor logistic models by reason of the foregoing discussion are suitable for this period, but the following relationship appears feasible as the following analysis illustrates.

$$\frac{dN}{dt} = AN^x \qquad \text{A and x constant.}$$

This differential equation can be solved by separation of variables giving

$$N = \left[(1 - x)At + N_0^{1-x}\right]^{\frac{1}{1-x}} \qquad \text{where } N = N_0 \text{ when } t = 0 \qquad (1)$$

If for $t_0 = 0$, t_1 and t_2, $N = N_0$, N_1 and N_2 respectively then x is found by the relationship

$$\frac{N_1^{1-x} - N_0^{1-x}}{N_2^{1-x} - N_0^{1-x}} = \frac{t_1}{t_2} \qquad (2)$$

and from this it is clear

$$A = \frac{N_1^{\,1-x} - N_0^{\,1-x}}{(1 - x)\,t_1} \tag{3}$$

The doubling time for this model can be found as follows:

Let D be the time required at time t for the population to double

Then

$$\frac{N(t + D)}{N(t)} = 2$$

giving for this particular model

$$\frac{\left[(1 - x)\ A(t + D) + N_0^{\,1-x}\right]^{\frac{1}{1-x}}}{\left[(1 - x)\ At + N_0^{\,1-x}\right]^{\frac{1}{1-x}}} = 2$$

Solving this relationship in the usual manner yields

$$D = \frac{\left[2^{1-x} - 1\right]\left[(1-x)At + N_0^{\,1-x}\right]}{(1 - x)A} \tag{4}$$

The values of the constants A and x vary the behaviour of this model considerably and consequently before any further analysis can occur the values of A and x for this period, need to be calculated.

Using the following initial and boundary conditions (e.g. system initialized at 1650 A.D.)

$$t = 0 \quad N = 545 \times 10^{6}$$

$$t = 150 \quad N = 906 \times 10^{6}$$

$$t = 325 \quad N = 4000 \times 10^{6}$$

expression (2) becomes

$$\frac{(906)^{1-x} - (545)^{1-x}}{(4000)^{1-x} - (545)^{1-x}} = \frac{150}{325} \qquad (5)$$

which can be solved to yield $x = 2.04$.

If this x value is then substituted into expression (3) A is found to be 2.12×10^{-12}.

When these values for A and x are substituted in expression (1) the relationship for the population growth can be expressed explicitly as:

$$N(t) = (-2.2 \times 10^{-12}t + 8.21 \times 10^{-10})^{-.96}$$

Table 7.2 shows the predicted population values of this model and compares these values to those observed. Due to the nature of expression (5) a rounding error occurs when x is calculated, and this causes a slight error in the reproduction of the initial and boundary conditions.

TABLE 7.2

Year	Population (in millions)	
	Data	Predicted
1650	545	528
1750	728	714
1800	906	870
1850	1171	1114
1900	1608	1560
1950	2400	2610
1976	4000	4000

If the substitution for A and x are made in expression (4) then the doubling time of this model is

$$D = \frac{(2^{-1.04} - 1)\ (-1.04 \times 2.12 \times 10^{-12}t + (545 \times 10^{6})^{-1.04})}{-1.04 \times 2.12 \times 10^{-12}}$$

$$= -.5t + 186$$

When $t = 0$ (i.e. 1650) the doubling time is 186 years. From Table 7.2 it can be seen that the observed data shows a doubling time of approximately 200 years. Similarly for $t = 200$ (i.e. 1850) the doubling time is 86 years which closely approximates the doubling time of the estimated data.

Finally, consider the expression

$$N(t) = (-2.2 \times 10^{-12}t + 8.21 \times 10^{-10})^{-.96}$$

It is clear that as t approaches 374 the population tends toward infinity while for t greater than 374 the population begins to decrease. In terms of calendar years this critical time is 2024 A.D. Obviously, the actual population will not become infinite in this time. However, the closeness of fit of this model to the real world suggests that unless a change in growth patterns occurs some type of discontinuity will result and the predicted sharpness of this discontinuity suggests it will be unpleasant in nature.

Due to the different growth patterns of European and non-European settlements (as discussed in Chapter IV) it is necessary to consider the average life expectancies of these areas separately.

As Ansley Coale (p. 22) points out, the average life expectancy of the European settlements in the period 1650 - 1850 was certainly no higher than 35 years and probably less than this in certain areas. The United Nations has estimated the average life expectancy in the non-European settlements to be 32 years as recently as 3 decades ago. Bearing in mind the life expectancy of 20 years in the period 0 A.D. - 1650 A.D. it appears reasonable to assume an average life expectancy of 30 years for both European and non-European settlements in the period 1650 - 1850.

From 1850 to 1940 the average life expectancy of non-European settlements remained constant at 32 years (Ansley Coale p.25) but the average life expectancy of the European settlements rose by 23 years (U.N. Pop'n. Bulln. p. 54). After this period the European life expectancy rose to its current value of 70 years while the non-European life expectancy rose some 18 years in a short period of only 30 years.

If the life expectancy of European and non-European settlements is taken as 50 years and 32 years respectively in 1900 then, given the relative population sizes of 573×10^6 and 1035×10^6 the following weighted average can be considered to be the average life expectancy of the population as a whole.

$$A.L.E._{1900} = \frac{573 \times 50 + 1035 \times 32}{1608}$$

$$= 38 \text{ years}$$

Similarly the average life expectancy of the population in 1950 is calculated by

$$A.L.E._{1950} = \frac{900 \times 55 + 1500 \times 32}{2500}$$

$$= 39 \text{ years.}$$

For the purpose of calculating the cumulative population, an average life expectancy of 39 years will be used for the period 1850 - 1950 as the above averages show the rapidly increasing numbers in the non-European settlements have offset the rapid gains in life expectancy of 50 years will be assumed for the population as a whole.

Thus the cumulative population from 1650 - 1976 can be found by:

$$C.P. = \frac{1}{30}\int_{0}^{200} N(t)\,dt + \frac{1}{38}\int_{200}^{300} N(t)\,dt + \frac{1}{50}\int_{300}^{325} N(t)\,dt$$

$$= 10.78 \times 10^{9}$$

Hence, the cumulative population for the period 1650 - 1976 is calculated as 10.78 billion.

CHAPTER VIII

CONCLUSION

CONCLUSION

If the cumulative populations proposed by the three models are now added together a total cumulative population of 86 billion results for the period 1,000,000 B.C. - 1976. It is interesting now to compare this figure with that obtained from other models and this comparison can be seen in Table 8.1

TABLE 8.1

CUMULATIVE POPULATION (in billions)

	Keyfitz	Wellemeyer	Deevey	Winkler	Present Study
1,000,000 B.C. - 8000 B.C.	14	12	66	-	11
8000 B.C. - 1650 A.D.	38	42	-	-	64.3
1650 - 1976	18+	23+	-	-	10.7
TOTAL	70+	77+	110+	5300+	86

It should be noted before proceeding that the "+" sign in the above table refers to the fact that the figures quoted for these models are slightly low due to the estimates being made as far back as 1960.

The 5300 billion proposed by Winkler appears far too high, and as Keyfitz (1966, p. 582) points out, the compound interest formula used in this model will produce many vastly differing results if time periods of differing lengths are used. The four other models produce remarkably close estimates of the total population, and in particular the model of Keyfitz closely approximates the model proposed in this thesis if more realistic average life expectancies are used rather than using the figure of 25 years for the entire period.

The closeness of these models is particularly interesting when it is observed that they are all, except for the present study, based on an exponential relationship, the unsuitability of which has been discussed in earlier chapters. Consequently, one might suggest that the cumulative population was suspected of being close to 80 billion and the models were constructed with this in mind. The model proposed by this thesis not only produced a cumulative population which appears to fall in the generally accepted area, but arrives at this figure by considering the social, economic and evolutionary history of man.

The close tie between the history of man and his numbers will always make estimates of future population size, except in the short run, little more than educated guesses unless some time in the future man learns to control his destiny. However, one trend does seem to emerge from the results of this thesis. The average population growth per year for the period 1,000,000 B.C. - 8000 B.C. was 1100 and this increased to 6.7 million for the period 8000 B.C. - 1650 A.D. During the last period this rose to 33 million and must be considerably higher at this time. There seems little doubt that this trend will continue for the next ten years at least and the implications of this to a world of finite resources are not likely to be pleasant.

The above notwithstanding, the progress of man from stone axe to nuclear power in a relatively short span of time represents a story of great achievement, and one cannot help but feel that from the 86 billion who have ever lived will come the solution to the population crisis.

APPENDIX A

The following is a minor generalisation of Kendall's birth, death and migration model which appears in Pollard (p. 65). The initial condition of zero population at time $t = 0$ has been altered to allow for an initial population of N_o individuals where $N_o > 0$.

Given an initial population N_o, birth rate λ, death rate μ and immigration rate ν then:

1. The probability of exactly one birth in time dt is $n\lambda dt + o(dt)$ {n is popl'n. size at time t}
2. Each existing individual at time t has a chance $\mu dt + o(dt)$ of dying in the time interval dt.
3. The probability of exactly one immigrant in time dt is $\nu dt + o(dt)$.

The probability of a population of n individuals in the time period $t + dt$ is given by:

$$P_n(t + dt) = P_{n-1}(t)\ (n - 1)\lambda dt + P_{n-1}(t)\nu dt + P_{n+1}(t)(n+1)\mu dt + P_n(t)\ \{1 - [n(\lambda + \mu) + \nu]dt\} \qquad \{A1\}$$

Where $P_{n-1}(t)(n-1)\lambda dt$ is the probability of n-1 people at time t and one birth occurring in time interval t+dt; $P_{n-1}(t)\nu dt$ is the probability of (n-1) people at time t and one immigrant in time $t + dt$; $P_{n+1}(t)(n+1)\mu dt$ is the probability of (n+1) people at time t and one death in time interval t+dt and $P_n(t)\{1 - [n(\lambda+\mu) + \nu]dt\}$ is the probability of n people at time t and no births, deaths or immigrations in time period $t + dt$.

$$\therefore P'_n(t) = [(n - 1)\lambda + \nu]P_{n-1}(t) + P_{n+1}(t)\ (n+1)\mu - \{n(\lambda + \mu) + \nu\}P_n(t)$$

To find the mean population, $M(t)$, at time t, multiply equation {A1} by n and sum from n = 0 to infinity.

$$\text{i.e.}\quad \sum_{n=0}^{\infty} nP'_n(t) = \sum_{n=0}^{\infty}\{n(n-1)\lambda + n\nu\}\, P_{n-1}(t) + \sum_{n=0}^{\infty} n(n+1)\mu\, P_{n+1}(t) - \sum_{n=0}^{\infty}\{n^2(\lambda+\mu) + n\nu\}\, P_n(t)$$

$$\text{then}\quad {}^{d}/_{dt}\sum_{n=0}^{\infty} n\, P_n(t) = \sum_{n=0}^{\infty} (\lambda - \mu) n\, P_n(t) + \nu$$

$$\text{giving}\quad {}^{d}/_{dt}\, M(t) = (\lambda - \mu)\, M(t) + \nu \qquad \text{where} \sum_{n=0}^{\infty} nP_n(t) = M(t) \quad \{A2\}$$

Using the integrating factor $e^{-(\lambda - \mu)t}$ equation{A2} becomes:

$${}^{d}/_{dt}\,\{M(t)e^{-(\lambda-\mu)t}\} = \nu e^{-(\lambda-\mu)t}$$

$$\text{and hence}\quad M(t) = \frac{\nu}{\lambda-\mu}\{e^{(\lambda-\mu)t} - 1\} + N_0 e^{(\lambda-\mu)t}$$

To determine the distribution of the population size at time t consider the probability generating function:

$$\phi(z,t) = \sum_{n=0}^{\infty} P_n(t)\, z^n$$

In particular from equation{A1}

$$\frac{\partial\phi}{\partial t} = \sum_{n=0}^{\infty} P'_n(t)\, z^n$$

$$= \sum_{n=0}^{\infty}\{(n-1)\lambda + \nu\}P_{n-1}(t)z^n + \sum_{n=0}^{\infty}(n+1)\mu\, P_{n+1}(t)z^n$$

$$- \sum_{n=0}^{\infty}\{n(\lambda+\mu) + \nu\}\, P_n(t)\, z^n$$

$$= \sum_{n=0}^{\infty}\{(\lambda z - \mu)(z-1)\}\, nP_n(t)\, z^{n-1} + \sum_{n=0}^{\infty} \nu(z-1)\, P_n(t)\, z^n$$

$$= (\lambda z - u)(z-1)\frac{\partial\phi}{\partial z} + \nu(z-1)\,\phi(z,t) \qquad \{A3\}$$

The auxiliary pair of equations (Keyfitz p. 364) for the above linear partial differential equation is

$$- dt = \frac{dz}{(\lambda z-\mu)(z-1)} = \frac{-d\phi}{\nu(z-1)\phi}$$

solving $-dt = \dfrac{dz}{(\lambda z-\mu)(z-1)}$ yields

$$C = \{\frac{z-\mu}{z-1}\} e^{-(\lambda-\mu)t} \quad \text{where C is a constant}$$

then solving $\dfrac{dz}{(\lambda z-\mu)(z-1)} = \dfrac{-d\phi}{\nu(z-1)\phi}$ gives

$$K = (\lambda z - \mu)\phi^{\lambda/\nu}$$

As K and C both depend upon the particular curve being considered K must be a function of C.

$$\text{Let} \quad K = \Phi\,(C)$$

$$\text{then} \quad (\lambda z-\mu)\phi^{\lambda/\nu} = \Phi\,\{\frac{(\lambda z-\mu)}{z-1} e^{-(\lambda-\mu)t}\} \qquad \{A4\}$$

Now at time $t = 0$ the population is N_o which implies $\phi(\underline{z},0) = z^{N_o}$

Substituting these values into equation{A4} results in

$$(\lambda z -\mu) z^{N_o\lambda/\nu} = \Phi\{\frac{\lambda z-\mu}{z-1}\} \qquad \{A5\}$$

Writing $L = \dfrac{\lambda z-\mu}{z-1}$ for the argument of Φ in equation {A5},z in terms of L becomes:

$$z = \frac{L-\mu}{L-\lambda}$$

Substituting this value for z into equation {A5} shows the form of the function as

$$\Phi\ (L) = \{\frac{\lambda(L-\mu)}{(L-\lambda)} - \mu\}\ \{\frac{L-\mu}{L-\lambda}\}^{N_o{}^{\lambda}/\nu}$$

Now equation {A4} becomes

$$(\lambda z - \mu)\ \phi^{\lambda}/_{\nu} = \{\ \frac{(\lambda z-\mu)e^{-(\lambda-\mu)t}}{z-1}(\lambda-\mu) \Big/ \frac{(\lambda z-\mu)\ e^{-(\lambda-\mu)t}}{z-1} - \lambda\}\quad X$$

$$\{\ \frac{(\lambda z-\mu\ )\ e^{-(\lambda-\mu)t}}{z-1} - \mu \Big/ \frac{(\lambda z-\mu\ )e^{-(\lambda-\mu)t}}{z-1} - \lambda\}^{N_o{}^{\lambda}/_{\nu}}$$

$$= \{\frac{(\lambda z-\mu)(\lambda-\mu)}{\lambda z-\mu-\lambda(z-1)e^{(\lambda-\mu)t}}\}\ \{\frac{(\lambda z-\mu)\ -\mu(z-1)\ e^{(\lambda-\mu)t}}{(\lambda z-\mu)\ -\lambda\ (z-1)\ e^{(\lambda-\mu)t}}\}^{N_o{}^{\lambda}/_{\nu}}$$

$$\therefore\quad \phi(z,t) = \{\frac{\lambda-\mu}{\lambda z-\mu-\lambda(z-1)e^{(\lambda-\mu)t}}\}^{\nu/_{\lambda}}\ \{\frac{(\lambda z-\mu)-\mu(z-1)e^{(\lambda-\mu)t}}{(\lambda z-\mu)\ -\ \lambda(z-1)e^{(\lambda-\mu)t}}\}^{N_o}$$

APPENDIX B

MEAN LIFETIME CALCULATION

Let $\mu(x)\,dx$ be the probability of an individual aged x dying in the next dx

$l(x)$ is the fraction of individuals aged 0 surviving to at least age x.

Then $\frac{l(x + dx)}{l(x)}$ is the probability of an individual aged x surviving to at least age x + dx

$$\text{i.e.} \quad \frac{l(x + dx)}{l(x)} = 1 - \mu(x)\,dx$$

$$\Rightarrow \quad \frac{l(x + dx) - l(x)}{dx} = -\mu(x)\; l(x)$$

$$\Rightarrow \quad \frac{d}{dx}\,(l(x)) = -\mu(x)\,l(x)$$

$$\Rightarrow \quad l(x) = e^{-\int \mu(x)\,dx} \qquad \text{where } l(0) = 1$$

Now define f(x) as the fraction of individuals aged 0 who will die in the period x to x + dx

$$\text{Then } l(x) = \int_x^{\infty} f(x)\; dx \qquad \text{and } \frac{d}{dx}\,(l(x)) = -f(x)$$

The mean lifetime can then be defined as follows:-

$$\text{Mean lifetime} = \int_0^{\infty} x\, f(x)\, dx$$

$$= -\int_0^{\infty} x \frac{d}{dx}(l(x))\,dx$$

$$= \Big[-x\, l(x))\Big]_0^{\infty} + \int_0^{\infty} l(x)\, dx$$

$$= \int_0^{\infty} l(x)\, dx$$

$$= \int_0^{\infty} e^{-\mu(x)\,dx}\, dx$$

When $\mu(x)$ is a constant

$$\text{then M.L.} = \int_0^{\infty} e^{-\mu x}\, dx$$

$$= \frac{1}{\mu}$$

Thus, once the death rate is fixed, the value of the average life expectancy is also fixed. When the birth and death rates are almost equal, as was the case in Chapter V, the value of the birthrate also follows.

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